Breadcrumbs: Adversarial Class-Balanced Sampling for Long-tailed Recognition

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1. Proof of Lemma 1

Lemma 1. *Consider the augmentation of $\mathcal{Z}_y^e$ with the snapshot transferred from epoch $e' < e$ by EMANATE, i.e. $\mathcal{A}_y^e = \mathcal{Z}_y^e \cup \mathcal{Z}_y^{e' \to e}$, where $\mathcal{Z}_y^{e' \to e}$ is as defined in (6, paper). Then*

$$L_y(\mathcal{A}_y^e, \mathbf{W}^e, \mathbf{b}^e) - L_y(\mathcal{Z}_y^e, \mathbf{W}^e, \mathbf{b}^e) \geq \frac{L_y(\mathcal{Z}_y^{e'}, \mathbf{W}^{e'}, \mathbf{b}^{e'}) - L_y(\mathcal{Z}_y^e, \mathbf{W}^e, \mathbf{b}^e)}{2}, \quad (1)$$

where $(\mathbf{W}^e, \mathbf{b}^e)$ is the classifier of (10, paper).

Proof. From (9, paper),

$$\begin{aligned} & L_y(\mathcal{A}_y^e, \mathbf{W}^e, \mathbf{b}^e) = \\ & = \frac{1}{2} L_y(\mathcal{Z}_y^e, \mathbf{W}^e, \mathbf{b}^e) + \frac{1}{2} L_y(\mathcal{Z}_y^{e' \to e}, \mathbf{W}^e, \mathbf{b}^e) \end{aligned}$$

and since

$$\begin{aligned} & L_y(\mathcal{Z}_y^{e' \to e}, \mathbf{W}^e, \mathbf{b}^e) = L_y(\{\mathbf{z}_i^{e'} - \bar{\mathbf{z}}^{e'} + \bar{\mathbf{z}}^e\}, \mathbf{W}^e, \mathbf{b}^e) \\ & = -\frac{1}{|\mathcal{Z}_y^{e'}|} \Sigma_i \log \nu_y(\mathbf{W}^e \mathbf{z}_i^{e'} - \mathbf{W}^e \bar{\mathbf{z}}^{e'} + \mathbf{W}^e \bar{\mathbf{z}}^{e'} + \mathbf{b}^e) \\ & = L(\mathcal{Z}_y^{e'}, \mathbf{W}^e, \mathbf{b}^e - \mathbf{W}^e \bar{\mathbf{z}}^{e'} + \mathbf{W}^e \bar{\mathbf{z}}^e) \end{aligned}$$

it follows from (10, paper) that

$$L_y(\mathcal{Z}_y^{e' \to e}, \mathbf{W}^e, \mathbf{b}^e) \geq L(\mathcal{Z}_y^{e'}, \mathbf{W}^{e'}, \mathbf{b}^{e'})$$

and

$$L_y(\mathcal{A}_y^e, \mathbf{W}^e, \mathbf{b}^e) \geq \frac{L_y(\mathcal{Z}_y^e, \mathbf{W}^e, \mathbf{b}^e) + L(\mathcal{Z}_y^{e'}, \mathbf{W}^{e'}, \mathbf{b}^{e'})}{2}$$

from which the lemma follows. $\square$